

the $\mathcal{H}^1$ -norm, and $\mathcal{H}^1$ -convergence of $\mathbf{u}_\varepsilon$ to $\mathbf{u}$ follows from the L^2 -convergence of $\mathbf{u}_\varepsilon$ to $\mathbf{u}$ and the L^2 -convergence of $\nabla \mathbf{u}_\varepsilon$ to $\nabla \mathbf{u}$.

For the L^2 -convergence of $\mathbf{u}_\varepsilon$ to $\mathbf{u}$, we use the following lemma. For $\mathbf{u} \in \mathbf{H}^1(\Omega)$, we define $\mathbf{u}_\varepsilon$ as the solution of the problem

$$\begin{cases} \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon) = \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}) & \text{in } \Omega, \\ \mathbf{u}_\varepsilon = 0 & \text{on } \partial\Omega, \end{cases} \quad (2.10)$$

where $\mathbf{A}_\varepsilon$ is the matrix-valued function defined by $\mathbf{A}_\varepsilon(\mathbf{x}) = \mathbf{A}(\mathbf{x}/\varepsilon)$. The following lemma is due to Ciorara and Murat [10].

Lemma 2.1. *Let $\mathbf{u} \in \mathbf{H}^1(\Omega)$ and let $\mathbf{u}_\varepsilon$ be the solution of (2.10). Then, $\mathbf{u}_\varepsilon \rightarrow \mathbf{u}$ in $L^2(\Omega)$ as $\varepsilon \rightarrow 0$.*

For the L^2 -convergence of $\nabla \mathbf{u}_\varepsilon$ to $\nabla \mathbf{u}$, we use the following lemma. For $\mathbf{u} \in \mathbf{H}^1(\Omega)$, we define $\mathbf{u}_\varepsilon$ as the solution of the problem

$$\begin{cases} \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon) = \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}) & \text{in } \Omega, \\ \mathbf{u}_\varepsilon = 0 & \text{on } \partial\Omega, \end{cases} \quad (2.11)$$

where $\mathbf{A}_\varepsilon$ is the matrix-valued function defined by $\mathbf{A}_\varepsilon(\mathbf{x}) = \mathbf{A}(\mathbf{x}/\varepsilon)$. The following lemma is due to Ciorara and Murat [10].

Lemma 2.2. *Let $\mathbf{u} \in \mathbf{H}^1(\Omega)$ and let $\mathbf{u}_\varepsilon$ be the solution of (2.11). Then, $\nabla \mathbf{u}_\varepsilon \rightarrow \nabla \mathbf{u}$ in $L^2(\Omega)$ as $\varepsilon \rightarrow 0$.*

For the L^2 -convergence of $\mathbf{u}_\varepsilon$ to $\mathbf{u}$, we use the following lemma. For $\mathbf{u} \in \mathbf{H}^1(\Omega)$, we define $\mathbf{u}_\varepsilon$ as the solution of the problem

$$\begin{cases} \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon) = \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}) & \text{in } \Omega, \\ \mathbf{u}_\varepsilon = 0 & \text{on } \partial\Omega, \end{cases} \quad (2.12)$$

where $\mathbf{A}_\varepsilon$ is the matrix-valued function defined by $\mathbf{A}_\varepsilon(\mathbf{x}) = \mathbf{A}(\mathbf{x}/\varepsilon)$. The following lemma is due to Ciorara and Murat [10].

Lemma 2.3. *Let $\mathbf{u} \in \mathbf{H}^1(\Omega)$ and let $\mathbf{u}_\varepsilon$ be the solution of (2.12). Then, $\mathbf{u}_\varepsilon \rightarrow \mathbf{u}$ in $L^2(\Omega)$ as $\varepsilon \rightarrow 0$.*

For the L^2 -convergence of $\nabla \mathbf{u}_\varepsilon$ to $\nabla \mathbf{u}$, we use the following lemma. For $\mathbf{u} \in \mathbf{H}^1(\Omega)$, we define $\mathbf{u}_\varepsilon$ as the solution of the problem

$$\begin{cases} \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon) = \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}) & \text{in } \Omega, \\ \mathbf{u}_\varepsilon = 0 & \text{on } \partial\Omega, \end{cases} \quad (2.13)$$

where $\mathbf{A}_\varepsilon$ is the matrix-valued function defined by $\mathbf{A}_\varepsilon(\mathbf{x}) = \mathbf{A}(\mathbf{x}/\varepsilon)$. The following lemma is due to Ciorara and Murat [10].

Lemma 2.4. *Let $\mathbf{u} \in \mathbf{H}^1(\Omega)$ and let $\mathbf{u}_\varepsilon$ be the solution of (2.13). Then, $\nabla \mathbf{u}_\varepsilon \rightarrow \nabla \mathbf{u}$ in $L^2(\Omega)$ as $\varepsilon \rightarrow 0$.*

For the L^2 -convergence of $\mathbf{u}_\varepsilon$ to $\mathbf{u}$, we use the following lemma. For $\mathbf{u} \in \mathbf{H}^1(\Omega)$, we define $\mathbf{u}_\varepsilon$ as the solution of the problem

$$\begin{cases} \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}_\varepsilon) = \operatorname{div}(\mathbf{A}_\varepsilon \nabla \mathbf{u}) & \text{in } \Omega, \\ \mathbf{u}_\varepsilon = 0 & \text{on } \partial\Omega, \end{cases} \quad (2.14)$$

where $\mathbf{A}_\varepsilon$ is the matrix-valued function defined by $\mathbf{A}_\varepsilon(\mathbf{x}) = \mathbf{A}(\mathbf{x}/\varepsilon)$. The following lemma is due to Ciorara and Murat [10].

Lemma 2.5. *Let $\mathbf{u} \in \mathbf{H}^1(\Omega)$ and let $\mathbf{u}_\varepsilon$ be the solution of (2.14). Then, $\mathbf{u}_\varepsilon \rightarrow \mathbf{u}$ in $L^2(\Omega)$ as $\varepsilon \rightarrow 0$.*

For the L^2 -convergence of $\nabla \mathbf{u}_\varepsilon$ to $\nabla \mathbf{u}$, we use the following lemma. For $\mathbf{u} \in \mathbf{H}^1(\Omega)$, we define $\mathbf{u}_\varepsilon$ as the solution of the problem